Analysis and Design of an Integrated Quadrature Mixer with Improved Noise, Gain, and Image Rejection

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Abstract

The analysis of a quadrature mixer based on the CMOS double-balanced mixer is presented. The noise, image rejection, conversion gain, and linearity of the quadrature mixer are compared to that of a Gilbert mixer under square wave and sinusoidal LO drive. It is shown that the quadrature mixer exhibits a 3 dB improvement in noise and conversion gain, and improved image rejection, compared to a pair of Gilbert mixers when the LO signal is a large amplitude sine wave.

1 Introduction

The mixer is an ubiquitous component of wireless systems. An ideal mixer multiplies the signal at the radio-frequency (RF) port with a signal at the local-oscillator (LO) port to create the intermediate-frequency (IF) signal. If the RF and LO signals are sinusoids, it is clear that the IF signal has components at two frequencies. There is a high-frequency component at the sum of the RF and LO frequencies, and a low frequency signal at the difference of the RF and LO signals. Therefore, a mixer can effect up-conversion (in a transmitter) or down-conversion (in a receiver).

The mixer generally follows the low-noise amplifier and therefore the performance of the mixer very much determines the performance of the overall system. It is desired that a mixer has high linearity so as to contribute minimally to intermodulation and desensitization. It is also desired that the mixer has a low noise figure so as to contribute little to the noise figure of the overall system. It is also usually advantageous for the mixer to have some gain to minimize the contribution of the IF circuit noise to the overall noise figure of the system. Finally, of course, it is desired that the mixer consumes the minimum power possible. In a quadrature system, it is additionally required that there is little mismatch in amplitude or phase response between the I and Q channels.

A commonly used mixer in CMOS systems is the Gilbert mixer, shown in Figure 1. This mixer is made of a differential transistor, formed by M1 and M2, and a pair of mixer cores, formed by M3-M4 and M5-M6. The transistor shown is a differential pair, but grounded-source transistors could be used as well. The mixer cores are each essentially a differential pair, and therefore switch the input current into a pair of differential outputs. The amount of current that goes to each output depends on the input voltage to the mixer core. In effect, the mixer core generates a differential output current that is approximately proportional to the input current multiplied by the differential signal at the mixer core gates. This current is usually driven into resistive loads to form an output voltage.

The above is true as long as the differential pairs that comprise the mixer cores remain in their linear region. When the LO signal is large, all of the input current is switched to one output, and no further increase in the LO signal can affect the output current. This is called the commutating mode of the mixer; in effect, the output current is proportional to the input current multiplied by a clipped version of the LO signal. If the LO signal is large enough, this mode approximately multiplies the input current by a square wave.

Figure 1: Schematic of a MOS Gilbert mixer

When downconversion occurs, two signals are mixed to the same IF frequency. The signal at the LO frequency plus the IF frequency, and the signal at the LO frequency minus the IF frequency, are both mixed to the IF frequency. One of these signals is the desired signal, and the other is called the image signal. The image signal and the desired signal are indistinguishable after downconversion, leading to corruption of the desired signal. A common way of avoiding this image problem in fully-integrated receivers is the use of quadrature downconversion. In quadrature downconversion, the RF signal is multiplied by both the in-phase (I) LO signal and a quadrature (Q) LO signal. In effect, the input signal is multiplied by a complex exponential. In this manner the image and desired signals remain distinct in the complex IF signal. In order to maintain this image rejection, a high degree of amplitude and phase matching between the I and Q channels is required [1].

In order to form a quadrature mixer, two Gilbert mixers with the same RF input and quadrature LO signals are often used. The mismatch between the mixers then creates amplitude and phase mismatch between the generated IF signals, leading to loss of image rejection. If the mixers are operated in the commutating mode, the effect of mismatch in the mixer core is usually negligible, because each transistor operates approximately as a switch. However, the mismatch due to the in the transconductors can be significant.

2 Quadrature Mixer Analysis

The mixer in Figure 2 was developed [2, 3] to reduce the loss of image rejection in a quadrature mixer. It is basically a pair of Gilbert mixers whose transconductor transistors have been shorted at the drain terminal. In this manner, there is no mismatch between channels due to the transconductor, because the transconductor is shared by the I and Q channels.

The operation of the quadrature mixer relative to a pair of Gilbert mixers depends on the LO signal used. Two modes of operation are considered in this paper: fully-commutating, and partially-commutating. The specifications of interest are the con-
version gain, the noise, the linearity, and the image rejection. The fully non-commutating mode, in which the LO amplitude is too small to bring the mixer core out of linear operation, is not discussed. In this mode, all specifications of interest are degraded, and it is therefore disadvantageous to operate in this mode.

In order to compare the performance of the quadrature mixer and a pair of Gilbert mixers, the quadrature mixer should have the same power consumption, input capacitance, and LO capacitance as the pair of Gilbert mixers. This is accomplished by making the width of the quadrature mixer transconductor transistors twice that of the Gilbert mixer transconductor transistors, and the bias current of the quadrature mixer twice that of each Gilbert mixer. The mixer cores of the quadrature mixer are made identical to those of the Gilbert mixers. This is consistent with the view of the quadrature mixer as a connected pair of Gilbert mixers. It is clear that the quadrature mixer has twice the transconductance of each Gilbert mixer. Such sizing and biasing is assumed throughout this paper.

Under all LO driving conditions, the linearity of the quadrature mixer is similar to that of a pair of Gilbert mixers. The dominant source of nonlinearity in a mixer is generally the transconductor. The linearity of short-channel MOSFETs [4] and differential pairs depends on the gate overdrive voltage, process parameters, and length of the transistors. The quadrature mixer transconductor transistors have the same length and approximately the same overdrive voltage as the Gilbert mixer transconductor transistors, and therefore exhibit similar linearity.

2.1 Fully Commutating Operation

In the fully commutating mode, the LO signal is a large amplitude square wave so that only one transistors in each mixer core is conducting at any given time. In this mode, the input current to the mixer core is multiplied by a square wave to form the output current.

The signal band voltage conversion gain of a Gilbert mixer is given by (1). The transconductance of the quadrature mixer is twice that of a Gilbert mixer; however, the current is split to two outputs at all times. Therefore, the signal band conversion voltage gain to each differential output of the quadrature mixer is the same as the pair of Gilbert mixers.

\[ A_v = \frac{2}{\pi} \cdot g_m \cdot R_L \]  

(1)

A detailed analysis of the noise of the quadrature mixer and a Gilbert mixer in the fully-commutating mode is given in [3]. The pertinent results are summarized here. In this analysis, flicker noise has been neglected, and it has been assumed that the mixer load consists of resistors.

In both the quadrature mixer and a Gilbert mixer pair, the loads contribute noise at IF, and the load noise is uncorrelated among the outputs. Since the total load of the quadrature mixer is the same as that of a pair of Gilbert mixers, the total noise due to the loads is also the same.

The noise injected by one transconductor transistor in a Gilbert mixer is transferred to the differential outputs in a correlated manner, and therefore noise voltages must be added rather than noise powers [5, 6]. Each transconductor transistor in the quadrature mixer injects noise onto both the I and Q outputs. The noise at the IF frequency in the I and Q channels is correlated, and noise voltages must be added rather than noise powers. However, in the same way that the image frequency is rejected, the transconductor noise in the image band is rejected. The transconductor transconductance transistors each have twice the Gm of the Gilbert mixer transconductor transistors. With the same input voltage, the quadrature mixer transconductor transistors carry twice the signal current, but only twice the squared noise current, as the corresponding transistors in the Gilbert mixers. Since a system using the quadrature mixer has the same signal transfer function as a system using a pair of Gilbert mixers, the quadrature mixer has a 3 dB advantage over a pair of Gilbert mixers in noise due to the transconductors.

With ideal switching, then noise from the mixer core in a Gilbert mixer is negligible. In the quadrature mixer, the switch noise is not negligible when the mixer is driven by a square LO, because two mixer core transistors at the drain of each transconductor transistor are conducting at any time. Therefore there is a low-impedance path for the mixer core noise current to reach the output. Noise from each mixer core transistor in the quadrature mixer is injected onto both I and Q outputs. For instance, noise from M3 is injected onto IF+ but not onto IF-. M3 injects the same noise power onto each of IF+ and IF-, and the noise is correlated with 90° relative phase. Due to the correlation between mixer core noise between the I and Q channels, the system output contains noise from only one sideband, but the noise in the other sideband has to be added as voltage.

The noise from the two Gilbert mixers required in a quadrature system is uncorrelated, but noise is contributed from both the image band and the signal band. The total output-referred noise of a system using a pair of Gilbert mixers is given in (2). The total output-referred noise of the quadrature mixer is given in (3).

\[ \overline{v_n^2} = 16kT R_L \left[ \frac{16}{3\pi^2} g_m \cdot R_L + 1 \right] \]

(2)

\[ \overline{v_n^2} = 16kT R_L \left[ \frac{4}{3\pi^2} \left( g_m + g_{m,LO} \right) R_L + 1 \right] \]

(3)
The noise performance of the quadrature mixer relative to that of a pair of Gilbert mixers depends on the relative sizes of the transconductor to the mixer core transistors. The quadrature mixer has a transconductor noise advantage but a mixer core noise disadvantage. The quadrature mixer and a pair of Gilbert mixers have the same noise performance in the fully-commutating mode when the overdrive voltage of the transconductor transistors is equal to the average overdrive voltage of the mixer core transistors. When mixer core devices are made small in order to minimize the LO-driver power dissipation, the noise performance of the quadrature mixer relative to a pair of Gilbert mixers improves.

Image rejection can be calculated from phase mismatch and amplitude mismatch, as shown in [1]. Mismatch in transconductor transistors leads to mismatch in input impedance and mismatch in transconductance. In most cases, the former effect leads predominantly to phase mismatch, while the latter effect leads mainly to amplitude mismatch. Voltage conversion gain depends linearly on $g_{m,RF}$, as shown in (1). Therefore, process variations that lead to a mismatch in $g_{m,RF}$ lead to a proportional amplitude mismatch in the outputs.

The quadrature mixer exhibits a loss of image rejection due to mismatch in the mixer core transistors. Because two transistors at the drain of each transconductor transistor are conducting at all times, the relative $g_{m}$ of those transistors determine the ratio of current routed to each output (I and Q). The Gilbert mixer does not exhibit this effect at full switching because nearly all current is routed to one output at any time. Therefore, the image rejection advantage of the quadrature mixer due to perfect transconductor matching is offset by the loss of image rejection due to mixer core mismatch.

2.2 Partially Commutating Operation

The mixers are partially commutating when driven by a sine wave whose amplitude is sufficient to drive the mixer cores out of their linear range. A usual approximation is that a large amplitude sine wave approximates a square wave; however, we will show that the quadrature mixer exhibits some advantageous properties when driven by a sine wave that it does not exhibit when driven by a square wave. Mixers are generally driven by large amplitude sine waves; therefore, the partially-commutating mode is the mode that would generally be seen.

When the quadrature mixer is partially commutating, the switching behavior is significantly different than when it is fully-commutating. A plot of the current through the mixer cores in the partially-commutating mode is given in Figure 3. It can be seen that the entire current available at the drain of a transconductor transistor is switched through each switch transistor in the quadrature mixer for one-quarter of the LO period. This is contrary to the fully-commutating case, where two transistors at each RF node are on at a given time. This full switching occurs because the drain of the transconductor transistor is pulled high when a mixer core transistor has a high overdrive voltage. For instance, when the I channel LO input is high, the Q channel LO input is passing through zero. Since M3 has a high overdrive voltage, the drain of M1 is pulled high. When this happens, a threshold voltage is not maintained across M7 and M8, and those transistors turn off. Therefore, only M3 is available to source the current required by M1, and all signal current passes to the output. Full commutation is therefore only possible when the switching is too rapid to allow the drains of the transconductor transistors to respond.

This full switching causes the gain of the quadrature mixer to be larger than that of a simple Gilbert mixer under the same LO drive. This increase in gain comes about because the input current in the quadrature mixer is multiplied by the waveform seen in Figure 3, which has the Fourier coefficients (4) given in (6), where $\alpha$ is the ratio of the pulse width to the period. In the simple Gilbert mixer, the input current is multiplied by a square wave, which has the Fourier coefficients given in (5). The pulse height is twice as high for the quadrature case as for the Gilbert case, because twice the bias current is available compared to the Gilbert mixer. The waveform of the quadrature mixer has a fundamental amplitude that is $\sqrt{2}$ times that of the Gilbert mixer, when the pulse widths of the quadrature and Gilbert mixers are one-quarter and one-half of the period, respectively. This is shown in Figure 4. The quadrature mixer therefore has a gain that is 3 dB higher than that of a corresponding Gilbert mixer.

$$V_o(t) = a_0 + \frac{1}{\pi} \sum_{n=1}^{\infty} \left[ a_n \cos(2\pi n f_{LO}) + b_n \sin(2\pi n f_{LO}) \right]$$

(4)

$$a_n = \frac{2h}{\pi n} \sin(\alpha \pi n) - \sin(\pi n)$$

(5)

$$a_0 = \frac{2h}{\pi n} \left[ \sin(\pi n |1 - \alpha|) - \sin(\pi n) + \sin(\pi n \alpha) \right]$$

(6)

When only one mixer core transistor at each transconductor transistor drain is conducting, the noise contributed to the output due to that mixer core transistor is negligible. Therefore, the mixer cores of both the quadrature mixer and the Gilbert mixer contribute noise only when switching. For large amplitudes, the switching period is a fraction of the total LO period. Therefore, the noise due to the switch transistors in both the quadrature and Gilbert mixers is small. The quadrature mixer exhibits less noise due to the transconductor than a pair of Gilbert mixers. Therefore, the quadrature mixer will have a 3 dB lower noise figure than a pair of corresponding Gilbert mixers when operated in this mode.

The mixer core transistors in the quadrature mixer operate approximately as switches in this mode. Since only one transistor
4 Conclusions

A quadrature mixer based on the CMOS Gilbert mixer has been analyzed under square and sinusoidal LO drive. The Gilbert mixers have been shown to operate similarly in both cases, while the quadrature mixer has been shown to have advantageous properties when driven by a sinusoidal LO that it does not exhibit when switched by a square wave.

- **Conversion Gain** The signal-band voltage conversion gain of the quadrature mixer and a pair of Gilbert mixers is the same when fully-commutating. The quadrature mixer has 3 dB higher gain than a pair of corresponding Gilbert mixers when the LO is a large amplitude sinusoid. This is due to the fact that all of the available current from each transconductor transistor in the quadrature mixer is switched through one mixer core transistor at a time. This available current is twice that which is switched through each Gilbert mixer core transistor.

- **Noise** In the fully-commutating mode, the quadrature mixer has the same noise due to the load as a pair of Gilbert mixers. The quadrature mixer exhibits a 3 dB advantage in noise due to the transconductor, but shows a disadvantage in the noise due to the mixer core transistors. Depending on which is the dominant noise, the quadrature mixer in the fully-commutating mode can exhibit an overall advantage or disadvantage over a pair of Gilbert mixers. When driven by a large sinusoidal LO, the quadrature mixer and the Gilbert mixer exhibit negligible noise due to mixer core transistors. The quadrature mixer retains its 3 dB advantage in noise due to the transconductor. Therefore the quadrature mixer exhibits a 3 dB advantage over a pair of Gilbert mixers when driven by a large amplitude sine wave.

- **Image Rejection** When fully-commutating, the advantage of perfect transconductor matching in the quadrature mixer is offset by the disadvantage of mixer core mismatch. Therefore, the image rejection is similar to that of a pair of Gilbert mixers in this mode. However, when driven by a large sinusoidal LO, the mixer core mismatch has negligible effect on the amplitude or phase of the output. In this mode, the image rejection can be considerably greater than a pair of corresponding Gilbert mixers.

References